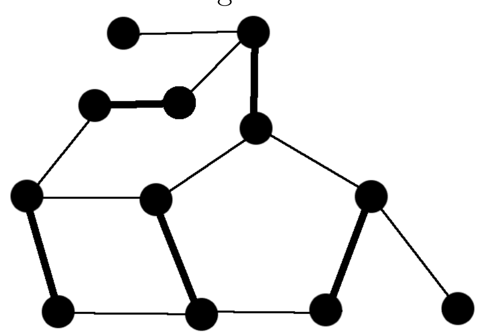
Graph Theory Midterm Practice Problems

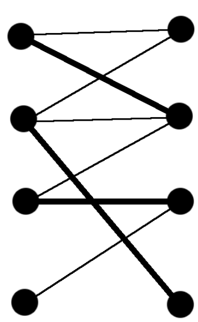
Use these as study aids in conjunction with the proofs on the homeworks and algorithms from the quizzes.

1. Consider the following in-degree and out-degree sequences for some hypothetical directed graph D. These sequences are not in any particular vertex order, so $S^+(1)$ and $S^-(1)$ don't necessarily refer to the same vertex. Is an Eulerian circuit possible on D? Justify your response.
   $S^+ = \{2, 4, 4, 2, 2, 6, 2, 8, 3\}, S^- = \{3, 2, 6, 2, 2, 4, 4, 2, 8\}$

2. Demonstrate a single iteration of our BFS-based Edmond's Blossom algorithm on the graph G below with the match given in bold. Show the M-augmenting path and the new larger match.



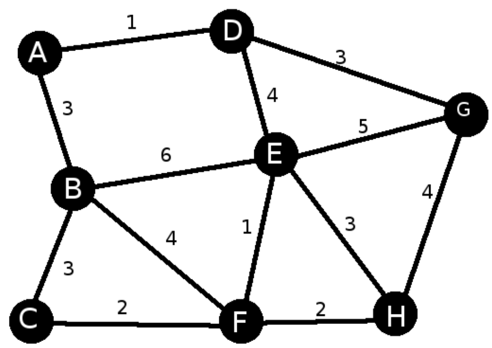
3. Demonstrate a single iteration of our M-augmenting paths algorithm for the bipartite graph below to increase the size of the match M given in bold on the graph below.



4. Directed graph D is weakly connected. There exists vertices $u, v \in V(D)$ such that a minimum u, v-cut partitions the graph into two vertex sets U and V, where $u \in U$ and $v \in V$. There are 6 directed edges from vertices in U to vertices in V and 5 directed edges from vertices in V to vertices in U. Give and justify a lowest possible upper bound on the directed edge-connectivity $\kappa'(D)$ of D.

5. Show the distance values from A for all vertices after processing each vertex with Djikstra's algorithm on the below graph. Also give the order of edges added to a

minimum spanning tree using Krushkal's algorithm on the below graph.



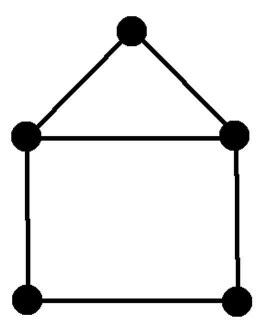
6. Draw a connected graph G with $|V(G)| = 6$ and $|M| = 2$, where M is a maximum match. Use Tutte's condition to prove that M is a maximum match.

7. Draw and gracefully label a connected graph of at least 4 vertices.

8. We have a bipartite graph $B_{X,Y}$ that has a minimum vertex cover C of size $|C| = 7$. We know that the sizes of each bipartite set are $|X| = 12$ and $|Y| = 8$. Place a tight bound on the possible size of a maximum match on B. Justify your response.

9. We have X, Y-bipartite graph $B_{X,Y}$ with $|X| = 9$ and $|Y| = 12$, where Hall's condition doesn't hold; i.e., $\exists S \subseteq X : |S| > |N(S)|$. However, we are able to create a maximal matching on B such that all $v \in X$ are saturated **except** for one single vertex u, where $u \in X$. Use this information to place tight upper and lower bounds on the sizes of possible vertex covers of B. Justify your response.

10. Use edge contraction to count how many possible spanning trees the following graph has: (5 pts)



11. Graph G has the following properties: maximum degree $\Delta(G) = 11$; minimum degree $\delta(G) = 4$; $\forall u, v \in V(G) : u, v$ lie on a common cycle. Put bounds on k for which G is possibly k-edge-connected based on these properties. Justify your response.

12. Prove that a degree sequence realizes a tree if and only if $d_i \geq 1, \sum_{i=1}^{n} d_i = (2n-2)$. Induction will help in one direction.